Practice 5-1

Find a quadratic model for each set of values.

1. \((-1, 1), (1, 1), (3, 9)\)

2. \((-4, 8), (-1, 5), (1, 13)\)

3. \((-10, 2), (2, 4), (3, -6)\)

4. 

\[
\begin{array}{c|c|c}
 x & 1 & 0 & 2 \\
 f(x) & 7 & -1 & 1 \\
\end{array}
\]

5. 

\[
\begin{array}{c|c|c|c}
 x & -4 & 0 & 1 \\
 f(x) & 1 & 9 & 16 \\
\end{array}
\]

6. 

\[
\begin{array}{c|c|c|c}
 x & -1 & 2 & 3 \\
 f(x) & 12 & 3 & 4 \\
\end{array}
\]

Identify the vertex and the axis of symmetry of each parabola.

7. 

8. 

9. 

Determine whether each function is linear or quadratic. Identify the quadratic, linear, and constant terms.

10. \(y = (x - 2)(x + 4)\)

11. \(y = 3x(x + 5)\)

12. \(y = 5x(x - 5) - 5x^2\)

13. \(f(x) = 7(x - 2) + 5(3x)\)

14. \(f(x) = 3x^2 - (4x - 8)\)

15. \(y = 3x(x - 1) - (3x + 7)\)

16. \(y = 3x^2 - 12\)

17. \(f(x) = (2x - 3)(x + 2)\)

18. \(y = 3x - 5\)

For each parabola, identify points corresponding to \(P\) and \(Q\).

19. 

20. 

21. 

22. A toy rocket is shot upward from ground level. The table shows the height of the rocket at different times.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (feet)</td>
<td>0</td>
<td>256</td>
<td>480</td>
<td>672</td>
<td>832</td>
</tr>
</tbody>
</table>

a. Find a quadratic model for this data.

b. Use the model to estimate the height of the rocket after 1.5 seconds.
Graph each function. If $a > 0$, find the minimum value. If $a < 0$, find the maximum value.

1. $y = -x^2 + 2x + 3$
2. $y = 2x^2 + 4x - 3$
3. $y = -3x^2 + 4x$
4. $y = x^2 - 4x + 1$
5. $y = -x^2 - x + 1$
6. $y = 5x^2 - 3$
7. $y = \frac{1}{2}x^2 - x - 4$
8. $y = 5x^2 - 10x - 4$
9. $y = 3x^2 - 12x - 4$

Graph each function.

10. $y = x^2 + 3$
11. $y = x^2 - 4$
12. $y = x^2 + 2x + 1$
13. $y = 2x^2 - 1$
14. $y = -3x^2 + 12x - 8$
15. $y = \frac{1}{2}x^2 + 2x - 1$

16. Suppose you are tossing an apple up to a friend on a third-story balcony. After $t$ seconds, the height of the apple in feet is given by $h = -16t^2 + 38.4t + 0.96$. Your friend catches the apple just as it reaches its highest point. How long does the apple take to reach your friend, and at what height above the ground does your friend catch it?

17. The barber’s profit $p$ each week depends on his charge $c$ per haircut. It is modeled by the equation $p = -200c^2 + 2400c - 4700$. Sketch the graph of the equation. What price should he charge for the largest profit?

18. A skating rink manager finds that revenue $R$ based on an hourly fee $F$ for skating is represented by the function $R = -480F^2 + 3120F$. What hourly fee will produce maximum revenues?

19. The path of a baseball after it has been hit is modeled by the function $h = -0.0032d^2 + d + 3$, where $h$ is the height in feet of the baseball and $d$ is the distance in feet the baseball is from home plate. What is the maximum height reached by the baseball? How far is the baseball from home plate when it reaches its maximum height?

20. A lighting fixture manufacturer has daily production costs of $C = 0.25n^2 - 10n + 800$, where $C$ is the total daily cost in dollars and $n$ is the number of light fixtures produced. How many fixtures should be produced to yield a minimum cost?

Graph each function. Label the vertex and the axis of symmetry.

21. $y = x^2 - 2x - 3$
22. $y = 2x - \frac{1}{4}x^2$
23. $y = x^2 + 6x + 7$
24. $y = x^2 + 2x - 6$
25. $y = x^2 - 8x$
26. $y = 2x^2 + 12x + 5$
27. $y = -3x^2 - 6x + 5$
28. $y = -2x^2 + 3$
29. $y = x^2 - 6$
Practice 5-3

Transforming Parabolas

Write the equation of the parabola in vertex form.

1. \( y = (x - 2)^2 - 3 \)
2. \( y = 6(x - 1)^2 + 4 \)
3. \( y = 2(x + 1)^2 + 1 \)

Graph each function.

7. \( y = (x - 2)^2 - 3 \)
8. \( y = (x - 6)^2 + 6 \)
9. \( y = \frac{1}{2}(x - 1)^2 - 1 \)

10. \( y = 8(x + 1)^2 - 2 \)
11. \( y = -3(x - 1)^2 + 3 \)
12. \( y = 3(x + 2)^2 + 4 \)

13. \( y = \frac{1}{8}(x + 1)^2 - 1 \)
14. \( y = \frac{1}{2}(x + 6)^2 - 2 \)
15. \( y = 2(x + 3)^2 - 3 \)

16. \( y = 4(x - 2)^2 \)
17. \( y = -2(x + 1)^2 - 5 \)
18. \( y = 4(x - 1)^2 - 2 \)

Write each function in vertex form.

19. \( y = x^2 + 4x \)
20. \( y = 2x^2 + 8x + 3 \)
21. \( y = -2x^2 - 8x \)

22. \( y = -x^2 + 4x + 4 \)
23. \( y = x^2 - 4x - 4 \)
24. \( y = x^2 + 5x \)

25. \( y = 2x^2 - 6 \)
26. \( y = -3x^2 - x - 8 \)
27. \( y = x^2 + 7x + 1 \)

28. \( y = x^2 + 8x + 3 \)
29. \( y = 2x^2 + 6x + 10 \)
30. \( y = x^2 + 4x - 3 \)

Identify the vertex and the y-intercept of the graph of each function.

31. \( y = 3(x - 2)^2 - 4 \)
32. \( y = -\frac{1}{3}(x + 6)^2 + 5 \)
33. \( y = 2(x - 1)^2 - 1 \)

34. \( y = \frac{2}{3}(x + 4)^2 - 3 \)
35. \( y = (x - 1)^2 + 2 \)
36. \( y = -3(x - 2)^2 + 4 \)

37. \( y = 4(x - 5)^2 + 1 \)
38. \( y = -2(x + 5)^2 - 3 \)
39. \( y = -5(x + 2)^2 + 5 \)
Practice 6-1

Find a cubic model for each function. Then use your model to estimate the value of \( y \) when \( x \neq 7 \).

1. \[
\begin{array}{c|c|c|c|c|c|c}
 x & 0 & 2 & 4 & 6 & 8 & 10 \\
\hline
 y & 25 & 21 & 20 & 23 & 19 & 17 \\
\end{array}
\]

2. \[
\begin{array}{c|c|c|c|c|c|c}
 x & 0 & 2 & 4 & 6 & 8 & 10 \\
\hline
 y & 3.1 & 4.2 & 4.3 & 4.4 & 5.1 & 6.7 \\
\end{array}
\]

Write each polynomial in standard form. Then classify it by degree and by number of terms.

3. \( 4x + x + 2 \)  
4. \( -3 + 3x - 3x \)  
5. \( 6x^4 - 1 \)  
6. \( 1 - 2s + 5s^4 \)  
7. \( 5m^2 - 3m^2 \)  
8. \( x^2 + 3x - 4x^3 \)  
9. \( -1 + 2x^2 \)  
10. \( 5m^2 - 3m^3 \)  
11. \( 5x - 7x^2 \)  
12. \( 2 + 3x^3 - 2 \)  
13. \( 6 - 2x^3 - 4 + x^3 \)  
14. \( 6x - 7x \)  
15. \( a^2(a^2 + a + 1) \)  
16. \( x(x + 5) - 5(x + 5) \)  
17. \( p(p - 5) + 6 \)  
18. \( (3c^2)^2 \)  
19. \( -(3 - b) \)  
20. \( 6(2x - 1) \)  
21. \( \frac{2}{3} + s^2 \)  
22. \( \frac{2x^4 + 4x - 5}{4} \)  
23. \( \frac{3 - \varepsilon^5}{3} \)

24. The lengths of the sides of a triangle are \( x + 4 \) units, \( x \) units, and \( x + 1 \) units. Express the perimeter of the triangle as a polynomial in standard form.

25. Find a cubic function to model the data below. (Hint: Use the number of years past 1940 for \( x \).) Then use the function to estimate the average monthly Social Security Benefit for a retired worker in 2010.

**Average Monthly Social Security Benefits, 1940–2003**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount (in dollars)</td>
<td>22.71</td>
<td>29.03</td>
<td>81.73</td>
<td>123.82</td>
<td>321.10</td>
<td>550.50</td>
<td>844.60</td>
<td>922.10</td>
</tr>
</tbody>
</table>

Source: www.infoplease.com

26. Find a cubic function to model the data below. (Hint: Use \( x \) to represent the gestation period.) Then use the function to estimate the longevity of an animal with a gestation period of 151 days.

**Gestation and Longevity of Certain Animals**

<table>
<thead>
<tr>
<th>Animal</th>
<th>Rat</th>
<th>Squirrel</th>
<th>Pig</th>
<th>Cow</th>
<th>Elephant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gestation (in days)</td>
<td>21</td>
<td>44</td>
<td>115</td>
<td>280</td>
<td>624</td>
</tr>
<tr>
<td>Longevity (in years)</td>
<td>3</td>
<td>9</td>
<td>10</td>
<td>12</td>
<td>40</td>
</tr>
</tbody>
</table>

Source: www.infoplease.com
Practice 6-2

Polynomials and Linear Factors

For each function, determine the zeros. State the multiplicity of any multiple zeros.

1. \( y = (x - 5)^3 \)
2. \( y = x(x - 8)^2 \)
3. \( y = (x - 2)(x + 7)^3 \)
4. \( f(x) = x^4 - 8x^3 + 16x^2 \)
5. \( f(x) = 9x^3 - 81x \)
6. \( y = (2x + 5)(x - 3)^2 \)

Write each function in standard form.

7. \( y = (x - 5)(x + 5)(2x - 1) \)
8. \( y = (2x + 1)(x - 3)(5 - x) \)

9. A rectangular box is 24 in. long, 12 in. wide, and 18 in. high. If each dimension is increased by \( x \) in., write a polynomial function in standard form modeling the volume \( V \) of the box.

Write a polynomial function in standard form with the given zeros.

10. \(-1, 3, 4\)
11. \(1, 1, 2\)
12. \(-3, 0, 0, 5\)
13. \(-2\) multiplicity 3

Write each expression as a polynomial in standard form.

14. \(x(x - 1)^2\)
15. \((x + 3)^2(x + 1)\)
16. \((x + 4)(2x - 5)(x + 5)^2\)

Write each function in factored form. Check by multiplication.

17. \( y = 2x^3 + 10x^2 + 12x \)
18. \( y = x^4 - x^3 - 6x^2 \)
19. \( y = -3x^3 + 18x^2 - 27x \)

Find the zeros of each function. Then graph the function.

20. \( y = (x + 1)(x - 1)(x - 3) \)
21. \( y = (x + 2)(x - 3) \)
22. \( y = x(x - 2)(x + 5) \)

Find the relative maximum, relative minimum, and zeros of each function.

23. \( f(x) = x^3 - 7x^2 + 10x \)
24. \( f(x) = x^3 - x^2 - 9x + 9 \)

Write each polynomial in factored form. Check by multiplication.

25. \( x^3 - 6x^2 - 16x \)
26. \( x^3 + 7x^2 + 12x \)
27. \( x^3 - 8x^2 + 15x \)

28. A rectangular box has a square base. The combined length of a side of the square base, and the height is 20 in. Let \( x \) be the length of a side of the base of the box.

a. Write a polynomial function in factored form modeling the volume \( V \) of the box.

b. What is the maximum possible volume of the box?
Practice 6-4

Solving Polynomial Equations

Factor the expression on the left side of each equation. Then solve the equation.

1. \(8x^3 - 27 = 0\)
2. \(x^3 + 64 = 0\)
3. \(2x^3 + 54 = 0\)
4. \(2x^3 - 250 = 0\)
5. \(4x^3 - 32 = 0\)
6. \(27x^3 + 1 = 0\)
7. \(64x^3 - 1 = 0\)
8. \(x^3 - 27 = 0\)
9. \(x^4 - 5x^2 + 4 = 0\)
10. \(x^4 - 12x^2 + 11 = 0\)
11. \(x^4 - 10x^2 + 16 = 0\)
12. \(x^4 - 8x^2 + 16 = 0\)
13. \(x^4 - 9x^2 + 14 = 0\)
14. \(x^4 + 13x^2 + 36 = 0\)
15. \(x^4 - 10x^2 + 9 = 0\)
16. \(x^4 + 3x^2 - 4 = 0\)
17. Over 3 yr, Lucia saved $550, $600, and $650 from baby-sitting jobs. The polynomial \(550x^3 + 600x^2 + 650x\) represents her savings, with interest, after 3 yr. The annual interest rate equals \(x - 1\). Find the interest needed so that she will have $2000 after 3 yr.

Solve each equation by graphing. Where necessary, round to the nearest hundredth.

18. \(2x^4 = 9x^2 - 4\)
19. \(x^2 - 16x = -1\)
20. \(6x^3 + 10x^2 + 5x = 0\)
21. \(36x^3 + 6x^2 = 9x\)
22. \(15x^4 = 11x^3 + 14x^2\)
23. \(x^4 = 81x^2\)
24. The product of three consecutive integers \(n - 1, n,\) and \(n + 1\) is \(-336\). Write and solve an equation to find the numbers.

Factor each expression.

25. \(x^3 - 125\)
26. \(x^4 - 8x^2 + 15\)
27. \(x^4 + x^2 - 2\)
28. \(x^3 + 1\)
29. \(x^4 - 2x^2 - 24\)
30. \(x^4 + 10x^2 + 9\)
31. \(x^3 + 27\)
32. \(x^4 + 7x^2 - 18\)

Solve each equation.

33. \(x^4 - x = 0\)
34. \(3x^4 + 18 = 21x^2\)
35. \(2x^4 - 26x^2 - 28 = 0\)
36. \(5x^4 + 50x^2 + 80 = 0\)
37. \(x^4 - 81 = 0\)
38. \(x^4 = 25\)
39. \(x^5 = x^3 + 12x\)
40. \(x^4 + 12x^2 = 8x^3\)